Beam divergence as a function of the Dihedral Angle offsets

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A dihedral angle offset produces a tilt in the exiting wavefront. In the geometrical optics solution, this produces a spot in the far field in the direction of the tilt. If the tilt is comparable to the diffraction spreading, the far field pattern is more complicated. It has to be computed by diffraction analysis. The beam divergence when all angles are offset by the same amount can be computed by Equation 1 (Arnold, David A., page 24 of SAO Special Report 382). This Equation was published by Rityn in 1967.

$$\gamma = \frac{4}{3}\sqrt{6}n\delta \qquad (1)$$

where,

γ = deviation of the exiting wavefront
n = index of refraction
δ = dihedral angle offset (the same for all back edges)

The formula can be rewritten as

$$\gamma = \frac{4}{3}\sqrt{6} = \sqrt{\frac{16}{9}6} = \sqrt{16\frac{2}{3}} = \sqrt{4\frac{8}{3}} \qquad (2)$$

I have written it in this form for comparison with the formulas for the deviation when only one or two dihedral angles are offset.

The spots in the far field come in pairs. The pairs correspond to opposite orders of reflection. The orders of reflection for the three sets of pairs are

123 and 321
213 and 312
312 and 213

If only one of the dihedral angles is offset, there are two spots in opposite directions. If two dihedral angles are offset there are 4 spots in the far field. One pair is bright. The other pair is weak. The two pairs are perpendicular to each other.

The deviation of the exiting wavefront for each case is listed below. The far field pattern is shown for each case. The pattern is computed for a 5 inch coated retroreflector. All the offsets are one arcsecond. The pattern is from -50 to 50 microradians in both the horizontal and vertical directions. The cross section is in million sq m. The offsets are listed for the X, Y, and Z dihedral angles of the retroreflector.

For 3 equal offsets in the three dihedral angles the beam divergence is

$$\gamma = \sqrt{4\frac{8}{3}}n\delta \tag{3}$$

The diffraction pattern is shown in Figure 1.

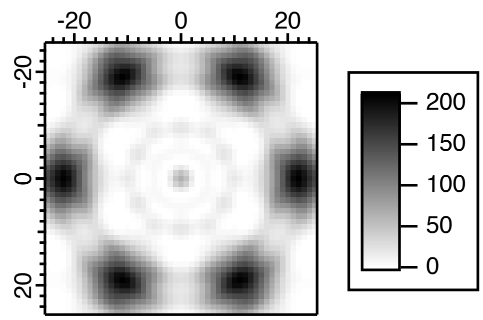

Figure 1

For two equal dihedral angle offsets, there are two values of the divergence. For the bright pair the divergence is

$$\gamma = \sqrt{3\frac{8}{3}}n\delta \tag{4}$$

For the weak pair the divergence is

$$\gamma = \sqrt{\frac{8}{3}}n\delta \tag{5}$$

The divergence of the two pairs of spots differs by a factor of $\sqrt{3}$ = 1.732.

The diffraction pattern with equal offsets in the X and Y dihedral angles is shown in Figure 2. The bright spots are aligned vertically parallel to the Z axis.

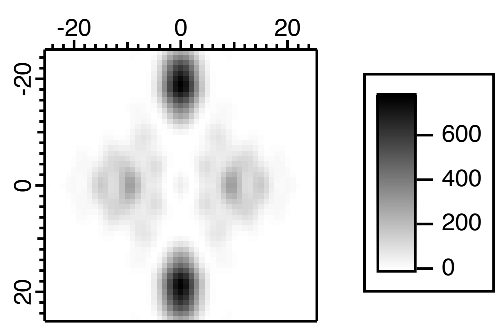

Figure 2

For a single dihedral angle offset there is only one pair of spots. The beam divergence is

$$\gamma = \sqrt{\frac{8}{3}}\, n\delta \qquad (6)$$

For a single offset, the divergence is smaller by a factor of $\sqrt{4}$ than with 3 offsets.

The diffraction pattern with a single offset in the Z dihedral angle is shown in Figure 3

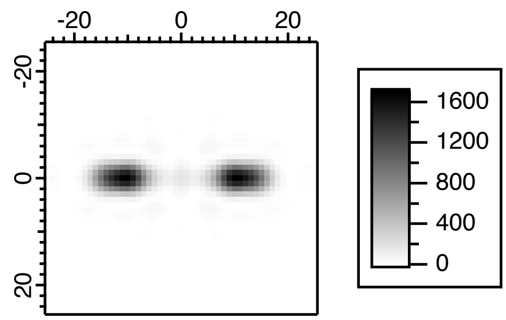

Figure 3.

For a single dihedral angle offset the dihedral angle δ required to give a beam spread γ is

$$\delta = \frac{\gamma}{\sqrt{\frac{8}{3}}\, n} \qquad (7)$$

Reference

Arnold, David A., page 24 of SAO Special Report 382,
http://davidarnoldresearch.org/1979SAO382.pdf