

**Subject:** Re: Letters for the LARES 2 design  
**Date:** Friday, June 2, 2017 at 8:37:38 PM Eastern Daylight Time  
**From:** David Arnold  
**To:** Ignazio Ciufolini, Reinhart Neubert  
**CC:** Ludwig Grunwaldt, Antonio Paolozzi, ErricosUmbric Pavlis

Dear Ignazio,

Below are my comments on the proposed design.

Best,

David Arnold

LARES-2  
1.0 inch diameter uncoated cube corners.

Cubes on the order of 1.5 inches have been used for a long time going back to the arrays placed on the moon. In recent times various missions have successfully used the smaller COTS cubes. Testing shows that these COTS cubes perform about as well as custom made cubes despite being about an order of magnitude cheaper. They are commercially available in standard sizes with no intentional dihedral angle offset.

The design being proposed for LARES-2 is not new. I originally proposed this design for LAGEOS-1 back in the 1970s. It was not used for two reasons: First, using 1.5 inch cubes met the 5 mm design goal. Second, using a large number of small cubes would significantly increase the cost. The cost is per cube rather than being proportional to size.

Today the accuracy requirement is 1 mm to achieve the goals of the terrestrial reference frame. Using the 1.5 inch cubes will not provide the increase in accuracy needed for present requirements. The satellite will have a very long lifetime. This design exceeds the present requirement by about a factor of two. This will enable the satellite to be useful over a longer period of time.

The diffraction pattern of a 1.5 inch cube is too narrow to account for velocity aberration. The desired beam width can be obtained by adding dihedral angle offsets. The width of the diffraction pattern is inversely proportional to the diameter of the cube. The desired beam spread to account for velocity aberration can be obtained by adjusting the size of the cube rather than adding a dihedral angle offset. Specifying a particular dihedral angle offset requires a custom made cube which is about an order of magnitude more expensive than COTS (Commercial Off The Shelf) cubes. Commercially available 1.0 inch diameter cubes with no intentional dihedral offset provide the required beam spread.

Using a larger number of small COTS cubes with no dihedral angle offset has the following advantages:

1. Thermal effects are a function of the size of the cube corner. The larger the cubes the longer the optical path length in the quartz. The longer optical path length results in larger phase gradients in the exiting wave front. This distorts the diffraction pattern. The effect is not linear with size. For example, for a linear phase gradient the phase difference is proportional to the square of the path length. The amplitude of the sum of two waves with a small phase difference depends on the cosine of the phase difference. The cosine is approximately quadratic for small angles. Even a small reduction in cube size from 1.5 to 1.0 inches should result in a significant improvement in the thermal stability.

2. The diffraction pattern created by using a dihedral angle offset is very irregular. This produces variations in the range correction and cross section within the velocity aberration annulus determined by the orbital velocity. Eliminating the offset produces a much smoother pattern.

3. In uncoated cubes there is an interaction between the polarization and the dihedral angle offset that causes an asymmetry in the diffraction pattern. This problem was noted in the design of the Lunar cubes. The asymmetry in the diffraction pattern is not accounted for in the analysis. This gives systematic errors in the range correction. Coated cubes do not show polarization effects. However, coated cubes have larger thermal gradients due to absorption of solar radiation by the metal coating on the back reflecting faces. This problem was identified during the study and testing of the cubes used on the moon.

4. The effect of manufacturing errors in the dihedral angles offsets is reduced by a factor of two by using a zero mean offset. For example an uncertainty of  $\pm 0.5$  arcsec has a maximum value of 0.5 arcsec if the average offset is zero. If the average offset is 1.0 arcsec the range is from 0.5 to 1.5 which is a change of 1.0 arcsec.

5. The asymmetric diffraction pattern due to a positive dihedral angle offset is rotated 90 degrees with respect to the pattern for a negative dihedral angle offset. The combination of random positive and negative offsets tends to circularize the pattern and reduce the asymmetry.

6. Using a larger number of small cubes reduces the variations in range correction and cross section as the incidence angle on the satellite changes. It is a better approximation to a uniform sphere with a constant cross section and range correction.

All of these advantages of using smaller cubes have been known for a long time and have been demonstrated by testing and/or analysis. The risk involved in using this new design is small compared to the advantages.