Dihedral angle offsets

A dihedral angle offset produces a tilt in the exiting wavefront. In the geometrical optics solution this produces a spot in the far field in the direction of the tilt. If the tilt is comparable to the diffraction spreading the far field pattern is more complicated and has to be computed by wave optics. The direction of the tilt can be computed by the formulas given below.

The formula given by Rityn (page 24 of SAO Special Report 382) is

$$\gamma = \frac{4}{3}\sqrt{6}n\delta$$

where,

γ = deviation of exiting wavefront
n = index of refraction
δ = dihedral angle offset (the same for all back angles)

The formula can be rewritten as

$$\frac{4}{3}\sqrt{6} = \sqrt{16\frac{6}{9}} = \sqrt{16\frac{2}{3}} = \sqrt{4\frac{8}{3}}$$

I have written it in this form for comparison with the formulas for the deviation when only one or two angles are offset.

The spots in the far field come in pairs. The pairs correspond to opposite orders of reflection and the pairs are opposite each other in the far field. The orders of reflection for the three sets of pairs are

123 and 321
213 and 312
312 and 213

When all three dihedral angle offsets are different, the three pairs are separate. For all angles equal, the pattern is a hexagon. For two angles the same and the third zero, two of the pairs overlap and the third is separate. The amplitude for the pairs that overlap is twice as great, and the intensity which is the square of the amplitude is 4 times as great as that of the single pair of points. For one angle offset with the other two offsets zero, all three pairs overlap so that only one pair of points is seen.

The deviation of the exiting wavefront for each case is

3 angles $\sqrt{4\frac{8}{3}}n\delta$

2 angles $\sqrt{3\frac{8}{3}}n\delta$, and $\sqrt{\frac{8}{3}}n\delta$

1 angle $\sqrt{\frac{8}{3}}n\delta$

For 2 angles, the two pairs of spots have different beam spreads. For only one angle, the beam spread is smaller by a factor of $\sqrt{4}$ = 2. For two angles, the pairs of spots differ by a factor of $\sqrt{3}$ = 1.732 in beam spread.
For one angle offset the dihedral angle δ required to give a beam spread γ is

$$\delta = \frac{\gamma}{\sqrt{\frac{8}{3}}n}$$

For γ 18 microradians we have

$$\delta = \frac{18}{\sqrt{\frac{8}{3}}n} = \frac{18}{1.633 \times 1.46} = 7.55 \mu rad = 1.557''$$

For an uncoated retroreflector, there are 6 spots around the central peak as a result of phase changes introduced by polarization effects during total internal reflection. With circular polarization, the spots are almost in the form of a hexagon. For linear vertical or horizontal polarization the positions of the six spots are slightly different. When a single dihedral angle offset is introduced, the phase front is tilted in a direction that falls between the six spots introduced by polarization effects. For this reason, adding a single dihedral angle does not produce two strong spots at the positions predicted by geometrical optics. The dihedral angle offset at first brightens two of the existing spots, but the effect maximizes around one arcsecond and is decreasing by the time the angle gets to 1.5 arseconds. In effect, the polarization and the dihedral angle offset are fighting each other with the dihedral angle trying to create a new spot in between the existing spots instead of reinforcing the existing spots. There is some gain, but not as much as might be expected and the average signal is decreasing as a result of adding dihedral angle offsets. The pattern produced with the dihedral angle offsets has a lot of structure and is not very symmetrical.